# Non-Markovian master equation for a system of Fermions interacting with an electromagnetic field

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## Abstract

For a system of charged Fermions interacting with an electromagnetic field, we derive a non-Markovian master equation in the second-order approximation of the weak dissipative coupling. A complex dissipative environment including Fermions, Bosons and the free electromagnetic field is taken into account. Besides the well-known Markovian term of Lindblad's form, that describes the decay of the system by correlated transitions of the system and environment particles, this equation includes new Markovian and non-Markovian terms proceeding from the fluctuations of the self-consistent field of the environment. These terms describe fluctuations of the energy levels, transitions among these levels stimulated by the fluctuations of the self-consistent field of the environment, and the influence of the time-evolution of the environment on the system dynamics. We derive a complementary master equation describing the environment dynamics correlated with the dynamics of the system. As an application, we obtain non-Markovian Maxwell-Bloch equations and calculate the absorption spectrum of a field propagation mode transversing an array of two-level quantum dots.

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## I. INTRODUCTION

Any realistic quantum mechanical system is open, being subjected to the influence of an uncontrollable environment of other particles that, generally, can not be separated from the structure of interest [1]. By neglecting the interaction with the environment, not only the accuracy of the results is altered, but also the internal consistency of the dynamical equations and the agreement of these equations with important physical principles are violated [2]. Essentially, physics of open systems means an approximate description of a complex system, where the total dynamics is reduced to that of a simpler part, called the system of interest, with correction terms for taking into account the effects of the other part, considered as the dissipative environment. This field of research has a long history, including more or less successful calculation schemes [3]. In such approximative calculations, important difficulties are encountered due to the complexity of the dissipative environment, on one hand, and, on the other hand, due to the quantum conditions, especially regarding the uncertainty relation and the positivity of the probabilities. Up to now, for the dynamic generator only two mathematical forms have been revealed as providing the positivity of the density matrix: the Liouvillian and the Lindbladian [4]. Using Lindblad's master equation, some basic physical systems have been studied: the harmonic oscillator with application to deep inelastic collisions [5] and nuclear giant resonances [6], quantum tunneling with application to cold fission [7], and the atom-field interaction with application to optical bistability [8]. In this framework the quantum dissipation is described by friction and diffusion components that, when some fundamental constraints are satisfied, are in full agreement with the quantum-mechanical principles.

However, Lindblad's master equation, now generally accepted in applications of major fields [9–11] has two important shortcomings: it is Markovian and contains a number of unspecified coefficients. Due to the non-Markovian evolution of an atom interacting with the electromagnetic field, important modifications of the absorption spectrum have been obtained by using Feynman's method of path integrals [12]. A non-Markovian master equation has been obtained for a one-dimensional system coupled to an environment of harmonic oscillators [13, 14]. It has been shown that the dissipative dynamics of such a system can be described by a master equation of the form originally derived in [5], but with coefficients depending on time [15, 16]. On the other hand, the quantum master equation of the harmonic oscillator, obtained in [5] in the axiomatic framework of the dynamical semigroups, has been reobtained on physical grounds, by considering an explicit Hamiltonian of the system interacting with the environment, and by using two approximations [17]: (1) a truncation of the density matrix series expansion in the approximation of the weak dissipative coupling, and (2) an averaging of the rapid oscillations induced by the coupling of the system with the environment.

We applied this calculation scheme to a system of charged Fermions in the second quantization, and derived a Markovian quantum master equation with explicit microscopic coefficients for the dissipative coupling with an environment of Fermions [18], Bosons [19], and free electromagnetic field [20]. We obtained a Lindbladian equation describing transitions $c_i^+ c_j$ of the system correlated to environment transitions $c_\beta^+ c_\alpha$ [2]:

$$\frac{d}{dt}\rho(t) = -\frac{i}{\hbar}[H^S, \rho(t)] + \sum_{ij} \lambda_{ij}([c_i^+ c_j \rho(t), c_j^+ c_i] + [c_i^+ c_j, \rho(t) c_j^+ c_i]), \tag{1}$$

where

$$H^S = H_0^S + V, \qquad H_0^S = \sum_i \varepsilon_i c_i^+ c_i \tag{2}$$

is the system Hamiltonian with an interaction term $V$. This equation has explicit microscopic coefficients

$$\lambda_{ij} = \lambda_{ij}^F + \lambda_{ij}^B + \gamma_{ij}, \tag{3}$$

including terms in agreement with the dynamical detailed balance principle, that for a rather low temperature $T \ll \varepsilon_{ji} \equiv \varepsilon_j - \varepsilon_i, j > i$, are of the form:

$$\lambda_{ij}^F = \frac{\pi}{\hbar}|\langle \alpha i | V^F | \beta j \rangle|^2 [1 - f_\alpha^F(\varepsilon_{ji})] g_\alpha^F(\varepsilon_{ji}) \qquad \text{- decay} \tag{4a}$$

$$\lambda_{ji}^F = \frac{\pi}{\hbar}|\langle \alpha i | V^F | \beta j \rangle|^2 f_\alpha^F(\varepsilon_{ji}) g_\alpha^F(\varepsilon_{ji}) \qquad \text{- excitation,} \tag{4b}$$

for an environment of Fermions with a density of states $g_\alpha^F(\varepsilon_\alpha)$ and a Fermi-Dirac distribution $f_\alpha^F(\varepsilon_\alpha)$,

$$\lambda_{ij}^B = \frac{\pi}{\hbar}|\langle \alpha i | V^B | \beta j \rangle|^2 [1 + f_\alpha^B(\varepsilon_{ji})] g_\alpha^B(\varepsilon_{ji}) \qquad \text{- decay} \tag{5a}$$

$$\lambda_{ji}^B = \frac{\pi}{\hbar}|\langle \alpha i | V^B | \beta j \rangle|^2 f_\alpha^B(\varepsilon_{ji}) g_\alpha^B(\varepsilon_{ji}) \qquad \text{- excitation,} \tag{5b}$$

for an environment of Bosons with a density of states $g_\alpha^B(\varepsilon_\alpha)$ and a Bose-Einstein distribution $f_\alpha^B(\varepsilon_\alpha)$, and

$$\gamma_{ij} = \frac{2\alpha}{c^2 \hbar^3} \vec{r}_{ij}^{\,2} \varepsilon_{ji}^3 (1 + \frac{1}{e^{\varepsilon_{ji}/T} - 1}) \tag{6}$$

for the free electromagnetic field coupled to the system of Fermions through a dipole moment $\vec{r}_{ij}$,where $\alpha$ is the fine structure constant $\alpha = \frac{e^2}{4\pi\varepsilon\hbar c}$. Eq. (1) takes into account the two-body potentials $V^F, V^B$, and the electric dipole potential in free electromagnetic field, that generally are stronger than the other many-body dissipative potentials. It is remarkable that although the dissipative terms of Eq. (1) have been derived in an approximation hierarchy, it has Lindblad's canonical form obtained as an exact expression from the semigroup theory.

In the above equation diagonalized single-particle states in the self-consistent field of the environment particles are considered without taking into account the fluctuations of this field. In this paper, we consider also these fluctuations, that determine fluctuations of the energy levels of the system and transitions among these levels. More than that, these fluctuations lead to a non-Markovian term in the second-order approximation of the reduced dynamics. Since we treat the problem in the weak coupling limit, the resultant master equation may not preserve the positivity of the density matrix, this equation being valid only for a rather short interval of time. In our approach, the time-evolution of the environment correlated with the time-evolution of the system is taken into account. While the system gets a new spatial distribution, the dissipative environment has a time-evolution including particle transitions and a modification of its own distribution according to the new distribution of the system. In Sec. II, we consider the density matrix of the total system, and derive the non-Markovian quantum master equation by tracing over the states of the environment with the condition of a quasi-continuum spectrum of these states. In Sec. III, by tracing the equation of the total density over the states of the system, we derive an evolution equation for the environment. This equation, describing the evolution of the environment, correlates with the evolution of the system to equilibrium and is in agreement with the dynamical detailed balance principle. Being derived in the second-order approximation, this equation is valid only for rather short intervals of time, otherwise violating the positivity of the density matrix. For a more realistic description of the environment evolution we add a term of dissipation to a bigger environment. In this case, the environment can no more get very far from equilibrium, the positivity of the density matrix of the environment being preserved. In Sec. IV, we consider the interaction with the electromagnetic field, and reduce the general quantum master equation for two components of the system: 1) the system of Fermions and 2) the electromagnetic field. We describe the non-Markovian dynamics of a many-level system interacting with a many-mode electromagnetic field. In

Sec. V we derive non-Markovian equations for slowly-varying amplitudes of a two-level system interacting with a single mode of the electromagnetic field. In Sec. VI we derive non-Markovian Maxwell-Bloch equations and calculate the dynamic absorption coefficient. In Sec. VII we consider an application and discuss the non-Markovian effects and in Sec. VIII we give conclusions.

## II. NON-MARKOVIAN MASTER EQUATION FOR FERMIONS

We consider a system with a density matrix $\rho(t)$, composed of a system of Fermions (S) with density matrix $\rho^S(t) = Tr_F\{\rho(t)\}$ and an electromagnetic field (F) with the density $\rho^F(t) = Tr_S\{\rho(t)\}$, in a complex dissipative environment (E) including Fermions, Bosons and the free electromagnetic field. If initially the environment is in the equilibrium state $R$, the initial density matrix $\chi(t)$ of the total system is of the form $\chi(0) = R\rho(0)$. We suppose that at the moment $t = 0$, due to the interaction $V$ of the system of Fermions with the electromagnetic field, or due to a non-equilibrium initial state $\rho(0) \neq \rho_T$, a time-evolution begins:

$$\frac{d}{dt}\tilde{\chi}(t) = -\frac{i}{\hbar}\left[\varepsilon\tilde{V}(t) + \varepsilon\tilde{V}^E(t), \tilde{\chi}(t)\right] \tag{7}$$

The density matrices $\tilde{\chi}(t)$ and $\tilde{\rho}(t)$ are expanded as follows:

$$\tilde{\chi}(t) = R\tilde{\rho}(t) + \varepsilon\tilde{\chi}^{(1)}(t) + \varepsilon^2\tilde{\chi}^{(2)}(t) + ... \tag{8a}$$

$$\frac{d}{dt}\tilde{\rho}(t) = \varepsilon\tilde{B}^{(1)}(\tilde{\rho}(t), t) + \varepsilon^2\tilde{B}^{(2)}(\tilde{\rho}(t), t) + ... . \tag{8b}$$

Besides the interaction potential $V$ of the system, the interaction potential $V^E$ of the system with the environment plays a central role. In these equations, tilde denotes operators in the interaction picture, e.g.

$$\tilde{\chi}(t) = e^{\frac{i}{\hbar}(H^E + H_0^S)}\chi(t)e^{-\frac{i}{\hbar}(H_0^S + H^E)}, \tag{9}$$

where $H^E$ is the Hamiltonian of the environment. The parameter $\varepsilon$ is introduced to handle the orders of the terms in this expansion, and is set to 1 in the final results. The reduced density of the system is

$$\tilde{\rho}(t) = Tr_E\{\tilde{\chi}(t)\}, \tag{10}$$

while the higher-order terms of the total density have the property:

$$Tr_E\{\tilde{\chi}^{(1)}\} = Tr_E\{\tilde{\chi}^{(2)}\} = ... \quad = 0. \tag{11}$$

The equilibrium density matrix of the environment

$$R = R^F \otimes R^B \otimes R^{FE} \tag{12}$$

includes the density matrix

$$R^F = \sum_{\alpha\beta} R^F_{\alpha\beta} c^+_\alpha c_\beta \tag{13}$$

for the Fermi component, and similar expressions for the Bose and the free electromagnetic field components. The perturbation term

$$V^E = \hbar \sum_{ij} \Gamma_{ij} c^+_i c_j \tag{14}$$

includes the operators

$$\Gamma_{ij} = \Gamma^F_{ij} + \Gamma^B_{ij} + \Gamma^{FE}_{ij} \tag{15}$$

with terms for the Fermion part of the environment

$$\Gamma^F_{ij} = \frac{1}{\hbar} \sum_{\alpha\beta} \langle \alpha i | V^F | \beta j \rangle c^+_\alpha c_\beta, \tag{16}$$

and similar expressions for the Boson part and for the free electromagnetic field. We notice that the coupling with the environment of the field component of the system of interest is not taken into account in this mathematical derivation. However, since the electromagnetic field is a system of quantum oscillators, in the following application this coupling will be treated according to the theory of the dissipative quantum oscillator [5]. Its mean-values are in agreement with the phenomenological electromagnetic theory. In the expression (8a) of the total dynamics, we distinguish the first term that describes the evolution of the system of interest while the state of the environment remains unchanged (Markov approximation). The higher-order terms of this series expansion describe an evolution of the environment correlated with the evolution of the system. In the second-order approximation, these terms are

$$\tilde{\chi}^{(1)}(t) = -\frac{i}{\hbar} R \int_0^t [\tilde{V}(t'), \tilde{\rho}(t')] dt' - i \sum_{ij} \int_0^t [\tilde{\Gamma}_{ij}(t') \tilde{c}^+_i(t') \tilde{c}_j(t'), R\tilde{\rho}(t')] dt' \tag{17a}$$

$$\tilde{\chi}^{(2)}(t) = -\frac{i}{\hbar} \int_0^t [\tilde{V}(t'), \tilde{\chi}^{(1)}(t')] dt' - -i \sum_{ij} \int_0^t [\tilde{\Gamma}_{ij}(t') \tilde{c}^+_i(t') \tilde{c}_j(t'), \tilde{\chi}^{(1)}(t')] dt', \tag{17b}$$

while for the reduced dynamics of the system we get the terms:

$$\tilde{B}^{(1)}[\tilde{\rho}(t),t] = -\frac{i}{\hbar}[\tilde{V}(t),\tilde{\rho}(t)] - i\sum_{ij} Tr_E[\tilde{\Gamma}_{ij}(t)\tilde{c}_i^+(t)\tilde{c}_j(t), R\tilde{\rho}(t)] \tag{18a}$$

$$\tilde{B}^{(2)}[\tilde{\rho}(t),t] = -i\sum_{ij} Tr_E[\tilde{\Gamma}_{ij}(t)\tilde{c}_i^+(t)\tilde{c}_j(t), \tilde{\chi}^{(1)}(t)]. \tag{18b}$$

In these equations we consider the reduced density matrix in the interaction picture $\tilde{\rho}(t)$ as slowly-varying in time. We assume the time-symmetry, that means that the time-integrals do not depend on the origin of time, but only on the relative intervals $t - t'$ between the actual time $t$ and the past time $t'$. With the transition operators in the interaction picture,

$$\tilde{c}_i^+(t)\tilde{c}_j(t) = e^{i\omega_{ij}t}c_i^+c_j, \qquad\qquad \omega_{ij} = \frac{\varepsilon_i - \varepsilon_j}{\hbar}, \tag{19}$$

we obtain:

$$\tilde{B}^{(1)}[\tilde{\rho}(t),t] = -\frac{i}{\hbar}[\tilde{V}(t),\tilde{\rho}(t)] - i\sum_{ij}\zeta'_{ij}[\tilde{c}_i^+(t)\tilde{c}_j(t),\tilde{\rho}(t)] \tag{20a}$$

$$\begin{aligned}\tilde{B}^{(2)}[\tilde{\rho}(t),t] = &\sum_{ij}\lambda_{ij}\left\{[c_i^+c_j\tilde{\rho}(t), c_j^+c_i] + [c_i^+c_j, \tilde{\rho}(t)c_j^+c_i]\right\} + \\ &+\sum_{ijkl}\zeta'_{ij}\zeta'_{kl}\int_0^t[\tilde{c}_i^+(t)\tilde{c}_j(t),[\tilde{c}_k^+(t')\tilde{c}_l(t'),\tilde{\rho}(t')]]dt'.\end{aligned} \tag{20b}$$

These equations include two families of dissipative coefficients: the matrix elements of the self-consistent field of the environment particles

$$\zeta'_{ij} = \frac{1}{\hbar Y^F}\int_{(\alpha)}\langle\alpha i|V^F|\alpha j\rangle f_\alpha^F(\varepsilon_\alpha)g_\alpha^F(\varepsilon_\alpha)d\varepsilon_\alpha \tag{21}$$

and the decay rates (3) with the terms (6) and

$$\lambda_{ij}^F = \frac{\pi}{\hbar Y^F}\int_{(\beta)}|\langle\alpha i|V^F|\beta j\rangle|^2[1 - f_\alpha^F(\varepsilon_\alpha)]f_\beta^F(\varepsilon_\beta)g_\alpha^F(\varepsilon_\alpha)g_\beta^F(\varepsilon_\beta)\Big|_{\omega_{\alpha\beta}=\omega_{ji}}d\varepsilon_\beta \tag{22a}$$

$$\lambda_{ij}^B = \frac{\pi}{\hbar Y^B}\int_{(\beta)}|\langle\alpha i|V^B|\beta j\rangle|^2[1 + f_\alpha^B(\varepsilon_\alpha)]f_\beta^B(\varepsilon_\beta)g_\alpha^B(\varepsilon_\alpha)g_\beta^B(\varepsilon_\beta)\Big|_{\omega_{\alpha\beta}=\omega_{ji}}d\varepsilon_\beta \tag{22b}$$

that for rather low temperatures reduce to the simple forms (4) and (5). In these expressions, $Y^F, Y^B$ represent the total numbers of the environment Fermions and Bosons, respectively. Although, by a diagonalizing the potential $V^F$ in (21) the transition matrix elements $\zeta'_{ij}$ vanish, such transitions continue to be generated by the thermal fluctuations of the environment particles over the states $|\alpha\rangle$. We take into account these fluctuations by considering in (21) the variances of the potential $V^F$:

$$\zeta'_{ij} \longrightarrow \zeta_{ij} = \frac{1}{\hbar}\sqrt{\frac{1}{Y^F}\int_{(\alpha)} \langle \alpha i|(V^F)^2|\alpha j\rangle f^F_\alpha(\varepsilon_\alpha) g^F_\alpha(\varepsilon_\alpha) d\varepsilon_\alpha}. \tag{23}$$

With (19) and using (8b) and (20) we obtain the non-Markovian quantum master equation:

$$\begin{aligned} \frac{d}{dt}\rho(t) = &-\frac{i}{\hbar}[H_0^S,\rho(t)] - \frac{i}{\hbar}[V,\rho(t)] - i\sum_{ij}\zeta_{ij}[c_i^+c_j,\rho(t)]+ \\ &+\sum_{ij}\lambda_{ij}([c_i^+c_j\rho(t),c_j^+c_i] + [c_i^+c_j,\rho(t)c_j^+c_i])+ \\ &+\sum_{ijkl}\zeta_{ij}\zeta_{kl}\int_0^t [c_i^+c_j, e^{-\frac{i}{\hbar}H_0^S(t-t')}[c_k^+c_l,\rho(t')]e^{\frac{i}{\hbar}H_0^S(t-t')}]dt'. \end{aligned} \tag{24}$$

In comparison with equation (1), the new equation (24) includes two new parts: a first-order Markovian part with the fluctuation matrix elements $\zeta_{ij}$ of the self-consistent field, and a non-Markovian part of the second-order in these elements. The Markovian part includes two families of terms where the diagonal elements $\zeta_{ii}$ describe fluctuations of the energy levels $\varepsilon_i$, and the non-diagonal elements $\zeta_{ij}$ random transitions of the system among these energy levels. The non-Markovian part describes memory effects due to the time-evolution of the self-consistent field of the environment.

In the next section we derive a dynamical equation for the environment.

## III. THE ENVIRONMENT DYNAMICS

The quantum master equation (24) describes the evolution of a system of Fermions under the action of the dissipative environment. For a complete description, we have also to calculate the environment evolution under the action of the system. In the second-order approximation, we obtain the environment density matrix by using (8a)

$$\tilde{\rho}^E(t) \equiv Tr_S\left\{\tilde{\chi}(t)\right\} = R + Tr_S\left\{\tilde{\chi}^{(1)}(t)\right\} + Tr_S\left\{\tilde{\chi}^{(2)}(t)\right\} + ... \tag{25}$$

From equations (7), (17) and (20) we get:

$$\frac{d}{dt}Tr_S\left\{\tilde{\chi}^{(1)}(t)\right\} = -i\sum_{ij} Tr_S\left[\tilde{\Gamma}_{ij}(t)\tilde{c}_i^+(t)\tilde{c}_j(t), R\tilde{\rho}(t)\right] \tag{26a}$$

$$\begin{aligned} \frac{d}{dt}Tr_S\left\{\tilde{\chi}^{(2)}(t)\right\} = &\sum_{ijkl}\int_0^t Tr_S\left\{\left[\tilde{\Gamma}_{ij}(t)\tilde{c}_i^+(t)\tilde{c}_j(t), R\zeta_{kl}\left[\tilde{c}_k^+(t')\tilde{c}_l(t'),\tilde{\rho}(t')\right]\right] - \right. \\ &\left. - \left[\tilde{\Gamma}_{ij}(t)\tilde{c}_i^+(t)\tilde{c}_j(t), \left[\tilde{\Gamma}_{kl}(t')\tilde{c}_k^+(t')\tilde{c}_l(t'), R\tilde{\rho}(t')\right]\right]\right\} dt'. \end{aligned} \tag{26b}$$

Here we consider the density matrix only in the interaction picture, since for the environment we are interested only in the slowly-varying part of the time evolution. Besides the equilibrium component $R$, the environment density matrix $\tilde{\rho}^E(t)$ is determined by three terms depending on the system density matrix $\tilde{\rho}(t)$ : (1) a term of the first-order in the system-environment correlations $\tilde{\Gamma}_{ij}(t)\tilde{c}_i^+(t)\tilde{c}_j(t)$, (2) a term of the second-order in the system-environment correlations and in the self-consistent field matrix elements, and (3) a term of the second-order in the system-environment correlations. We find that only the terms exclusively depending on the system-environment correlations and leading to Markovian terms, remain in the final result, while the terms depending on the self-consistent field reduce to zero.

With the explicit expressions of the time-dependent operators in the interaction picture, Eqs. (26) become:

$$\frac{d}{dt}Tr_S\left\{\tilde{\chi}^{(1)}(t)\right\} = -\frac{i}{\hbar}\sum_{ij}\sum_{\alpha\beta}\langle\alpha i|V^F|\beta j\rangle e^{i(\omega_{\alpha\beta}-\omega_{ji})t}[c_\alpha^+ c_\beta, R]\tilde{\rho}_{ji}(t) \tag{27a}$$

$$\frac{d}{dt}Tr_S\left\{\tilde{\chi}^{(2)}(t)\right\} = -\frac{1}{\hbar^2}\sum_{ij}\sum_{\alpha\beta}\int_0^t |\langle\alpha i|V^F|\beta j\rangle|^2 e^{i(\omega_{\alpha\beta}-\omega_{ji})(t-t')}\cdot$$
$$\cdot\left\{[c_\alpha^+ c_\beta, c_\beta^+ c_\alpha R]\tilde{\rho}_{ii}(t) - [c_\alpha^+ c_\beta, R c_\beta^+ c_\alpha]\tilde{\rho}_{jj}(t)\right\} dt' \tag{27b}$$

With the resonant condition and using Eq. (13) with the condition of a quasi-continuum spectrum of environment states, we obtain the environment master equation:

$$\begin{aligned}\frac{d}{dt}\tilde{\rho}^E(t) = &-\frac{i}{\hbar Y^F}\sum_{ij}\tilde{\rho}_{ji}(t)\int_{(\beta)}\langle\alpha i|V^F|\beta j\rangle g_\beta(\varepsilon_\beta)c_\alpha^+ c_\beta\cdot \\ &\cdot\{[1-f_\alpha(\varepsilon_\alpha)]\, f_\beta(\varepsilon_\beta) - [1-f_\beta(\varepsilon_\beta)]\, f_\alpha(\varepsilon_\alpha)\}\,|_{\omega_{\alpha\beta}=\omega_{ji}}\, d\varepsilon_\beta + \\ &+\frac{\pi}{\hbar Y^F}\sum_{ij}\int_{(\beta)}|\langle\alpha i|V^F|\beta j\rangle|^2 g_\alpha(\varepsilon_\alpha)g_\beta(\varepsilon_\beta)\left(c_\alpha^+ c_\alpha - c_\beta^+ c_\beta\right)\cdot \\ &\cdot\{[1-f_\alpha(\varepsilon_\alpha)]\, f_\beta(\varepsilon_\beta)\tilde{\rho}_{jj}(t) - [1-f_\beta(\varepsilon_\beta)]\, f_\alpha(\varepsilon_\alpha)\tilde{\rho}_{ii}(t)\}\,|_{\omega_{\alpha\beta}=\omega_{ji}}\, d\varepsilon_\beta\end{aligned} \tag{28}$$

This equation satisfies basic conditions and is very transparent to physical interpretations. The first term describes transitions $c_\alpha^+ c_\beta$ of the environment correlated with the system transitions of probabilities $\tilde{\rho}_{ji}(t)$. The transition probability from an initial state $|\beta\rangle$ to a final state $|\alpha\rangle$ is considered as the difference between the direct and the reverse transitions between these states, that depend on the occupation probabilities $f_\alpha(\varepsilon_\alpha)$ and $f_\beta(\varepsilon_\beta)$ and the probabilities $1-f_\alpha(\varepsilon_\alpha)$ and $1-f_\beta(\varepsilon_\beta)$ that these states are free. Since $\tilde{\rho}_{ji}(\infty) = 0$, this term vanishes for $t \longrightarrow \infty$. The second term describes population variations of the environment states while the system population takes non-equilibrium values. While the system population tends to the equilibrium values

$$\frac{\tilde{\rho}_{jj}(\infty)}{\tilde{\rho}_{ii}(\infty)} = \frac{[1 - f_\beta(\varepsilon_\beta)]\, f_\alpha(\varepsilon_\alpha)}{[1 - f_\alpha(\varepsilon_\alpha)]\, f_\beta(\varepsilon_\beta)} = e^{-(\varepsilon_\alpha - \varepsilon_\beta)/T}, \tag{29}$$

this term tends also to zero. That means that while the system tends to equilibrium according to the detailed balance principle, the environment state tends also to equilibrium. We also notice that the population variations of two states $|\alpha\rangle$ and $|\beta\rangle$ are opposite to one another, the total population being conserved. While the system is in an excited state, $\tilde{\rho}_{jj}(t) > \tilde{\rho}_{ii}(t)$, the higher states $|\alpha\rangle$ are stronger populated than the lower states $|\beta\rangle$.

We notice that Eq. (28), derived in the second-order approximation of the theory of perturbations, in a rather long interval of time has the tendency to violate the positivity of the density matrix. This equation of the environment, open only to the system of interest, has been written down only to illustrate the internal coherence of our theory. A more realistic equation for the environment could be written by openness to a bigger environment:

$$\begin{aligned}
\frac{d}{dt}\rho^E(t) = &-\frac{i}{\hbar}\left[H^E, \rho^E(t)\right] - \\
&- \frac{i}{\hbar Y^F} \sum_{ij} \rho_{ji}(t) \int_{(\beta)} \langle \alpha i | V^F | \beta j \rangle \left\{ [1 - f_\alpha(\varepsilon_\alpha)] f_\beta(\varepsilon_\beta) - [1 - f_\beta(\varepsilon_\beta)] f_\alpha(\varepsilon_\alpha) \right\} \cdot \\
&\qquad \cdot c_\alpha^+ c_\beta g_\beta(\varepsilon_\beta) d\varepsilon_\beta + \\
&+ \frac{\pi}{\hbar Y^F} \sum_{ij} \int_{(\beta)} |\langle \alpha i | V^F | \beta j \rangle|^2 \left\{ [1 - f_\alpha(\varepsilon_\alpha)] f_\beta(\varepsilon_\beta) \rho_{jj}(t) - [1 - f_\beta(\varepsilon_\beta)] f_\alpha(\varepsilon_\alpha) \rho_{ii}(t) \right\} \cdot \\
&\qquad \cdot (c_\alpha^+ c_\alpha - c_\beta^+ c_\beta) g_\alpha(\varepsilon_\alpha) g_\beta(\varepsilon_\beta) d\varepsilon_\beta + \\
&+ \sum_{\alpha\beta} \lambda_{\alpha\beta} \left( \left[ c_\alpha^+ c_\beta \rho^E(t), c_\beta^+ c_\alpha \right] + \left[ c_\alpha^+ c_\beta, \rho^E(t) c_\beta^+ c_\alpha \right] \right).
\end{aligned} \tag{30}$$

In this equation, the coupling with the bigger environment dominates the coupling with the system of interest while the positivity of the density matrix is preserved.

## IV. NON-MARKOVIAN MATTER-FIELD INTERACTION

In quantum optics, the matter-field dynamics is usually described as a rotating Bloch vector, or as an oscillating polarization coupled with a slowly-varying population described by Bloch-Feynman equations. In these equations originally derived for a closed system

[21], dissipation is usually introduced in terms of phenomenological quantities (e. g. [22]): the decay time of the population $T_1 \equiv 1/\gamma_{\|}$, and the incoherent relaxation time of the polarization $T_2 \equiv 1/\gamma_{\perp}$. Using the quantum master equation (24), we obtain a generalized description including also non-Markovian effects. In this equation we consider the interaction potential

$$V = -\frac{e\vec{A}\vec{p}}{M}, \tag{31}$$

where $M$ is the Fermion mass, $e$ is the electric charge. The momentum is given by

$$\vec{p} = iM\sum_{ij}\omega_{ij}\vec{r}_{ij}c_i^+c_j, \tag{32}$$

where $\vec{r}$ is the coordinate, while $\vec{A} = \sum_\nu \vec{A}_\nu$ is the potential vector of the electromagnetic field. We consider a weak-field approximation, when the square of the potential vector can be neglected in the Hamiltonian of the charged particle in the electromagnetic field. At the same time, we take the modes $\nu$ as plan waves, propagating in space with the wave vectors $\vec{k}_\nu$ :

$$\vec{A}_\nu = \frac{\hbar}{e}\vec{K}_\nu\left(a_\nu e^{i\vec{k}_\nu\vec{r}} + a_\nu^+ e^{-i\vec{k}_\nu\vec{r}}\right) \tag{33}$$

The coefficient

$$\vec{K}_\nu = \vec{1}_\nu\sqrt{\alpha\frac{\lambda_\nu}{\mathcal{V}}} \tag{34}$$

depends on the quantization volume $\mathcal{V}$, the wavelength $\lambda_\nu$, the polarization vector $\vec{1}_\nu$, and the fine-structure constant $\alpha$.

In a consistent description, an open system of Fermions can be coupled with an electromagnetic field only when this field is also open. Consequently, in the dissipative part of the master equation we consider also the Lindbladian terms of the field modes [5], here for $\nu$

$$\begin{aligned} L_\nu = & -i\frac{\Lambda_\nu}{2\hbar}\left([q_\nu, p_\nu\rho(t) + \rho(t)p_\nu] - [p_\nu, q_\nu\rho(t) + \rho(t)q_\nu]\right) - \\ & - \frac{D_{qq}^{(\nu)}}{\hbar^2}[p_\nu,[p_\nu,\rho(t)]] - \frac{D_{pp}^{(\nu)}}{\hbar^2}[q_\nu,[q_\nu,\rho(t)]] + \\ & + \frac{D_{pq}^{(\nu)}}{\hbar^2}\left([q_\nu,[p_\nu,\rho(t)]] + [p_\nu,[q_\nu,\rho(t)]]\right) \end{aligned} \tag{35}$$

described as harmonic oscillators with coordinates and momenta

$$q_\nu = \sqrt{\frac{\hbar}{2\omega_\nu}}\left(a_\nu^+ + a_\nu\right) \tag{36a}$$

$$p_\nu = i\sqrt{\frac{\hbar\omega_\nu}{2}}\left(a_\nu^+ - a_\nu\right), \tag{36b}$$

while $\Lambda_\nu$ and $D_{qq}^{(\nu)}$, $D_{pp}^{(\nu)}$, $D_{pq}^{(\nu)}$ represent friction and diffusion coefficients, respectively. In this way, we obtain the quantum master equation of the whole matter-field system

$$\begin{aligned} \frac{d}{dt}\rho(t) = -\frac{i}{\hbar}\sum_i \varepsilon_i[c_i^+c_i,\rho(t)] - i\sum_\nu \omega_\nu[a_\nu^+a_\nu,\rho(t)] + \sum_\nu \left\{L_{\nu+}[\rho(t)] + L_{\nu-}[\rho(t)]\right\} - \\ - \sum_{ij\nu}\omega_{ij}\vec{r}_{ij}\vec{K}_\nu\{[c_i^+c_ja_{\nu+},\rho(t)]e^{i\vec{k}_\nu\vec{r}} + [c_i^+c_ja_{\nu+}^+,\rho(t)]e^{-i\vec{k}_\nu\vec{r}} + \\ + [c_i^+c_ja_{\nu-},\rho(t)]e^{-i\vec{k}_\nu\vec{r}} + [c_i^+c_ja_{\nu-}^+,\rho(t)]e^{i\vec{k}_\nu\vec{r}}\} - \\ - i\sum_{ij}\zeta_{ij}[c_i^+c_j,\rho(t)] + \sum_{ij}\lambda_{ij}([c_i^+c_j\rho(t),c_j^+c_i] + [c_i^+c_j,\rho(t)c_j^+c_i]) + \\ + \sum_{ijkl}\zeta_{ij}\zeta_{kl}\int_0^t [c_i^+c_j, e^{-\frac{i}{\hbar}H_0^S(t-t')}[c_k^+c_l,\rho(t')]e^{\frac{i}{\hbar}H_0^S(t-t')}]dt'. \end{aligned} \tag{37}$$

with terms describing the interaction, decay, and fluctuations of this system, including non-Markovian effects. Since reflection phenomena are often important in a non-linear optical structure, we separated two modes of the electromagnetic field, designed by a plus and a minus sign and propagating in two opposite directions of a line, i.e. with the wave-vectors $\vec{k}_\nu$ and $-\vec{k}_\nu$.

In the mean-field approximation, we derive master equations for the two components of the matter-field system. Tracing over the field states, we obtain the quantum master equation of the system (S) of Fermions

$$\begin{aligned} \frac{d}{dt}\rho^S(t) = -\frac{i}{\hbar}\sum_i \varepsilon_i[c_i^+c_i,\rho^S(t)] - i\sum_{ij}\zeta_{ij}[c_i^+c_j,\rho^S(t)] + \\ + \frac{i}{2}\sum_{ij\nu}\frac{\omega_{ij}}{\omega_\nu}\vec{g}_{ij}\left(\vec{\mathcal{E}}_\nu(\vec{r},t)e^{-i\omega_\nu t} - \vec{\mathcal{E}}_\nu^*(\vec{r},t)e^{i\omega_\nu t}\right)[c_i^+c_j,\rho^S(t)] + \\ + \sum_{ij}\lambda_{ij}([c_i^+c_j\rho^S(t),c_j^+c_i] + [c_i^+c_j,\rho^S(t)c_j^+c_i]) + \\ + \sum_{ijkl}\zeta_{ij}\zeta_{kl}\int_0^t [c_i^+c_j, e^{-\frac{i}{\hbar}H_0^S(t-t')}[c_k^+c_l,\rho^S(t')]e^{\frac{i}{\hbar}H_0^S(t-t')}]dt'. \end{aligned} \tag{38}$$

The coupling coefficients are

$$\vec{g}_{ij} = \frac{e}{\hbar}\vec{r}_{ij} \tag{39}$$

and $\vec{\mathcal{E}}_\nu(\vec{r},t)$ are the amplitudes of the mean-values of the electric field

$$\langle\vec{E}_\nu\rangle = \frac{1}{2}\left[\vec{\mathcal{E}}_\nu(\vec{r},t)e^{-i\omega_\nu t} + \vec{\mathcal{E}}_\nu^*(\vec{r},t)e^{i\omega_\nu t}\right]. \tag{40}$$

The amplitude of the field includes the two counter-propagating components,

$$\vec{\mathcal{E}}_\nu(\vec{r},t) = \vec{\mathcal{E}}_{\nu+}(\vec{r},t)e^{i\vec{k}_\nu\vec{r}} + \vec{\mathcal{E}}_{\nu-}(\vec{r},t)e^{-i\vec{k}_\nu\vec{r}}, \tag{41}$$

with slowly-varying space amplitudes

$$\vec{\mathcal{E}}_{\nu+}(\vec{r},t) = 2i\frac{\hbar\omega_\nu}{e}\vec{K}_\nu\bar{a}_{\nu+}(\vec{r},t) \tag{42}$$

$$\vec{\mathcal{E}}_{\nu-}(\vec{r},t) = 2i\frac{\hbar\omega_\nu}{e}\vec{K}_\nu\bar{a}_{\nu-}(\vec{r},t) \tag{43}$$

as functions of the mean-values of the creation and annihilation operators

$$\langle a_{\nu+}\rangle = \bar{a}_{\nu+}(\vec{r},t)e^{-i\omega_\nu t}, \qquad \langle a_{\nu-}\rangle = \bar{a}_{\nu-}(\vec{r},t)e^{-i\omega_\nu t} \tag{44}$$

$$\langle a^+_{\nu+}\rangle = \bar{a}^*_{\nu+}(\vec{r},t)e^{i\omega_\nu t}, \qquad \langle a^+_{\nu-}\rangle = \bar{a}^*_{\nu-}(\vec{r},t)e^{i\omega_\nu t}. \tag{45}$$

We obtain the master equation of the density matrix of the field:

$$\begin{aligned}\frac{d}{dt}\rho^F(t) = &-i\sum_\nu \omega_\nu[a^+_{\nu+}a_{\nu+} + a^+_{\nu-}a_{\nu-}, \rho^F(t)] + \sum_\nu L_{\nu+}[\rho^F(t)] + \sum_\nu L_{\nu-}[\rho^F(t)] - \\ &- \sum_{ij\nu}\omega_{ij}\langle c^+_i c_j\rangle \vec{r}_{ij}\vec{K}_\nu \Big\{[a_{\nu+},\rho^F(t)]e^{i\vec{k}_\nu\vec{r}} + [a^+_{\nu+},\rho^F(t)]e^{-i\vec{k}_\nu\vec{r}} + \\ &+ [a_{\nu-},\rho^F(t)]e^{-i\vec{k}_\nu\vec{r}} + [a^+_{\nu-},\rho^F(t)]e^{i\vec{k}_\nu\vec{r}}\Big\}.\end{aligned} \tag{46}$$

We consider this master equation for a system of electrons (negative charge), and derive mean-field equations for the two propagation modes:

$$\frac{d}{dt}\langle a_{\nu+}\rangle = -i\omega_\nu\langle a_{\nu+}\rangle - \Lambda_{\nu+}\langle a_{\nu+}\rangle + \sum_{ij}\omega_{ij}\vec{r}_{ij}\vec{K}_\nu\rho^S_{ji}(\vec{r},t)e^{-i\vec{k}_\nu\vec{r}} \tag{47a}$$

$$\frac{d}{dt}\langle a^+_{\nu+}\rangle = i\omega_\nu\langle a^+_{\nu+}\rangle - \Lambda_{\nu+}\langle a^+_{\nu+}\rangle - \sum_{ij}\omega_{ij}\vec{r}_{ij}\vec{K}_\nu\rho^S_{ji}(\vec{r},t)e^{i\vec{k}_\nu\vec{r}}, \tag{47b}$$

and

$$\frac{d}{dt}\langle a_{\nu-}\rangle = -i\omega_\nu\langle a_{\nu-}\rangle - \Lambda_{\nu-}\langle a_{\nu-}\rangle + \sum_{ij}\omega_{ij}\vec{r}_{ij}\vec{K}_\nu\rho^S_{ji}(\vec{r},t)e^{i\vec{k}_\nu\vec{r}} \tag{48a}$$

$$\frac{d}{dt}\langle a^+_{\nu-}\rangle = i\omega_\nu\langle a^+_{\nu-}\rangle - \Lambda_{\nu-}\langle a^+_{\nu-}\rangle - \sum_{ij}\omega_{ij}\vec{r}_{ij}\vec{K}_\nu\rho^S_{ji}(\vec{r},t)e^{-i\vec{k}_\nu\vec{r}}. \tag{48b}$$

In these equations for the mean-values, only the friction parts in $\Lambda_{\nu+}$, $\Lambda_{\nu-}$ remain from the field Lindbladians $L_{\nu+}[\rho^F(t)]$, $L_{\nu-}[\rho^F(t)]$. The diffusion parts in $D^{(\nu)}_{qq}$, $D^{(\nu)}_{pp}$, $D^{(\nu)}_{pq}$ are responsible only for the field fluctuations, that are not treated here. With Eq. (40), these equations reduce to equations of the electric field amplitudes:

$$\frac{d}{dt}\vec{\mathcal{E}}_{\nu+}(\vec{r},t) = -\Lambda_\nu \vec{\mathcal{E}}_{\nu+}(\vec{r},t) + 2i\frac{\hbar\omega_\nu}{e}K_\nu^2 \sum_{ij} \omega_{ij}\vec{r}_{ij}\rho^S_{ji}(\vec{r},t)e^{i(\omega_\nu t - \vec{k}_\nu \vec{r})} \tag{49a}$$

$$\frac{d}{dt}\vec{\mathcal{E}}_{\nu-}(\vec{r},t) = -\Lambda_\nu \vec{\mathcal{E}}_{\nu-}(\vec{r},t) + 2i\frac{\hbar\omega_\nu}{e}K_\nu^2 \sum_{ij} \omega_{ij}\vec{r}_{ij}\rho^S_{ji}(\vec{r},t)e^{i(\omega_\nu t + \vec{k}_\nu \vec{r})}, \tag{49b}$$

where $\vec{r}$ in the exponential factors and in the slowly-varying amplitudes is the center of mass coordinate of the Fermi system with the dipole moment $\vec{r}_{ij}$. According to (34), the interaction terms in these equations are proportional to the system polarization terms $\frac{e\vec{r}_{ij}}{\mathcal{V}}$ of the system, that is in agreement with the classical model. The field equations (49) are coupled with the equations of the Fermi system by the matrix elements $\rho^S_{ij}(\vec{r},t)$. From the master equation (38) these equations are

$$\begin{aligned}\frac{d}{dt}\rho^S_{ji}(t) = &-i\omega_{ji}\rho^S_{ji}(t) - \frac{i}{2}\sum_k\sum_\nu \left[\vec{\mathcal{E}}_\nu(t)e^{-i\omega_\nu t} - \vec{\mathcal{E}}^*_\nu(t)e^{i\omega_\nu t}\right]\left[\frac{\omega_{jk}}{\omega_\nu}\vec{g}_{jk}\rho^S_{ki}(t) - \frac{\omega_{ki}}{\omega_\nu}\rho^S_{jk}(t)\vec{g}_{ki}\right] - \\ &- i\sum_k\left[\zeta_{jk}\rho^S_{ki}(t) - \rho^S_{jk}(t)\zeta_{ki}\right] + \sum_k\left[2\delta_{ji}\lambda_{jk}\rho^S_{kk}(t) - (\lambda_{kj}+\lambda_{ki})\rho^S_{ji}(t)\right] + \\ &+ \sum_{kl}\int_0^t \left\{\zeta_{jk}\left[\zeta_{kl}\rho^S_{li}(t') - \rho^S_{kl}(t')\zeta_{li}\right]e^{i\omega_{ik}(t-t')} - \left[\zeta_{jk}\rho^S_{kl}(t') - \rho^S_{jk}(t')\zeta_{kl}\right]\zeta_{li}e^{i\omega_{lj}(t-t')}\right\}dt'.\end{aligned} \tag{50}$$

Generally, the system of oscillating atomic variables

$$\rho^S_{ji}(\vec{r},t) \equiv \bar{\rho}^S_{ji}(\vec{r},t)e^{-i\omega_{ji}t} \tag{51}$$

with the amplitudes $\bar{\rho}^S_{ji}(\vec{r},t)$ generates the electromagnetic field in the whole spectrum of the transition frequencies $\omega_\nu \approx \omega_{ji}$ and of the higher-order harmonics.

Eqs. (50) coupled with the field equations (49) through the relation (41) describe the dissipative interaction of the field modes $\nu$ with a system of Fermions placed at the coordinate $\vec{r}$. In these equations, we distinguish three mechanism of dissipation: (1) dissipation of the field modes described by the field decay coefficients $\Lambda_\nu$, (2) the decay, described by the decay coefficients $\lambda_{ij}$, and (3) the the mean-field fluctuations of the environment particles, described by the matrix elements $\zeta_{ij}$. The decay coefficients $\lambda_{ij}$ describe correlated transitions of the system and environment particles, i.e. an energy exchange between the system and environment. Since $Tr\{\varepsilon_k c^+_k c_k[c^+_i c_j, \rho^S(t)]\} \equiv Tr\{\varepsilon_k[c^+_k c_k, c^+_i c_j]\rho^S(t)]\} = 0$, from (38) we find that the coefficients $\zeta_{ij}$ describe elastic processes, modifying the system evolution without changing the energy of the system. That means that the first-order terms in $\zeta_{ij}$ describe only the single-particle transitions of the system (S) induced by the mean-field fluctuations of the many-particle environment (E), and not the energy exchange between the two systems (S) and (E).

## V. ARRAY OF TWO-LEVEL QUANTUM DOTS IN RADIATION FIELD

We consider the problem of an array of two-level quantum dots situated at $z = 0$, as a two-dimensional system in the $xy$-plan, interacting with an electromagnetic mode with the quasi-resonant frequency $\omega \approx \omega_0 \equiv \omega_{10}$ that propagates in the direction $z$. We truncate the spectrum of the higher-order harmonics generated due to the nonlinearity of the system to the second order. In the interaction part of the field equations (49) we keep only the terms in the density matrix element $\rho^S_{10}(\vec{r}, t)$ that according to (51) are slowly varying in time, the other terms that are rapidly varying in time being neglected. Including the atomic detuning $\omega - \omega_0$ in the amplitude of this matrix element, in the second-order approximation of the Fourier series expansion the expression (51) takes a form

$$\rho^S_{10}(z,t) = \frac{1}{2}\left[\mathcal{S}_0(z,t) + \mathcal{S}(z,t)e^{-i\omega t} + \mathcal{S}_2(z,t)e^{-i2\omega t}\right] \tag{52}$$

with slowly-varying amplitudes for counter-propagating polarization waves

$$\begin{aligned} &\mathcal{S}_0(z,t), \\ &\mathcal{S}(z,t) = \mathcal{S}_+(z,t)e^{ikz} + \mathcal{S}_-(z,t)e^{-ikz}, \\ &\mathcal{S}_2(z,t) = \mathcal{S}_{2+}(z,t)e^{i2kz} + \mathcal{S}_{2-}(z,t)e^{-i2kz}. \end{aligned} \tag{53}$$

In Eq. (52), the term in $\mathcal{S}(z,t)$ corresponding to the injected field of frequency $\omega$ is dominant, while the other terms $\mathcal{S}_0(z,t)$ and $\mathcal{S}_2(z,t)$ generated only as nonlinear effects are much smaller. For the corresponding field amplitudes, we obtain the propagation equations

$$\frac{d}{dt}\mathcal{E}_+(z,t) = \frac{\partial}{\partial t}\mathcal{E}_+(z,t) + c\frac{\partial}{\partial z}\mathcal{E}_+(z,t) = -\Lambda_+\mathcal{E}_+(z,t) - iG\mathcal{S}_+(z,t) \tag{54a}$$

$$\frac{d}{dt}\mathcal{E}_-(z,t) = \frac{\partial}{\partial t}\mathcal{E}_-(z,t) - c\frac{\partial}{\partial z}\mathcal{E}_-(z,t) = -\Lambda_-\mathcal{E}_-(z,t) - iG\mathcal{S}_-(z,t) \tag{54b}$$

$$\frac{d}{dt}\mathcal{E}_{2+}(z,t) = \frac{\partial}{\partial t}\mathcal{E}_{2+}(z,t) + c\frac{\partial}{\partial z}\mathcal{E}_{2+}(z,t) = -\Lambda_{2+}\mathcal{E}_{2+}(z,t) - iG\mathcal{S}_{2+}(z,t) \tag{54c}$$

$$\frac{d}{dt}\mathcal{E}_{2-}(z,t) = \frac{\partial}{\partial t}\mathcal{E}_{2-}(z,t) - c\frac{\partial}{\partial z}\mathcal{E}_{2-}(z,t) = -\Lambda_{2-}\mathcal{E}_{2-}(z,t) - iG\mathcal{S}_{2-}(z,t) \tag{54d}$$

with the coupling coefficient

$$G = \frac{\hbar\omega_0}{2\varepsilon\mathcal{V}} g \ , \qquad g = \frac{e}{\hbar} r_{01} \ . \tag{55}$$

These field equations have non-zero polarizations only for $z = 0$, while for $z \neq 0$ they have only propagation and decay terms. From the $z$-dependence one obtains the absorption coefficient of the field transversing the quantum dot array, e.g

$$\alpha_+(t, z = 0) = -\frac{1}{|\mathcal{E}_+(t,z)|}\frac{\partial}{\partial z}|\mathcal{E}_+(t,z)|_{z=0}. \tag{56}$$

Evidently, the dissipative coefficients $\Lambda$ of these equations depend on the field mode.

To calculate the matrix elements (52) we particularize Eqs. (50) for a two-level system. We get equations of the non-diagonal elements $\rho^S_{10}(t) = \left[\rho^S_{01}(t)\right]^* \equiv \rho^S_{10}(z,t)\ |_{z=0}$:

$$\begin{aligned}
\frac{d}{dt}\rho^S_{10}(t) &= -[\gamma_\perp + i(\omega_0 + \zeta_{11} - \zeta_{00})]\rho^S_{10}(t) + i\big\{\zeta_{10} + \\
&+ \frac{g}{2}\big[\mathcal{E}(t)e^{-i\omega t} - \mathcal{E}^*(t)e^{i\omega t} + \frac{1}{2}\left(\mathcal{E}_2(t)e^{-i2\omega t} - \mathcal{E}^*_2(t)e^{i2\omega t}\right)\big]\big\}\big[\rho^S_{11}(t) - \rho^S_{00}(t)\big] + \\
&+ \int_0^t \big\{\big[2|\zeta_{10}|^2 + (\zeta_{11} - \zeta_{00})^2 e^{-i\omega_0(t-t')}\big]\rho^S_{10}(t') - 2\zeta^2_{10}\rho^S_{01}(t') - \\
&- \zeta_{10}(\zeta_{11} - \zeta_{00})\big[\rho^S_{11}(t') - \rho^S_{00}(t')\big]e^{-i\omega_0(t-t')}\big\}dt',
\end{aligned} \tag{57}$$

and equations for the diagonal elements:

$$\begin{aligned}
\frac{d}{dt}\rho^S_{11}(t) &= -i\frac{g}{2}\big[\mathcal{E}(t)e^{-i\omega t} - \mathcal{E}^*(t)e^{i\omega t} + \frac{1}{2}\left(\mathcal{E}_2(t)e^{-i2\omega t} - \mathcal{E}^*_2(t)e^{2i\omega t}\right)\big]\big[\rho^S_{10}(t) + \rho^S_{01}(t)\big] + \\
&+ i[\zeta_{01}\rho^S_{10}(t) - \rho^S_{01}(t)\zeta_{10}] - 2\big[\lambda_{01}\rho^S_{11}(t) - \lambda_{10}\rho^S_{00}(t)\big] + \\
&+ \int_0^t \big\{|\zeta_{01}|^2\big[e^{-i\omega_0(t-t')} + e^{i\omega_0(t-t')}\big]\big[\rho^S_{11}(t') - \rho^S_{00}(t')\big] - \\
&- (\zeta_{11} - \zeta_{00})\big[\zeta_{01}\rho^S_{10}(t')e^{-i\omega_0(t-t')} + \rho^S_{01}(t')\zeta_{10}e^{i\omega_0(t-t')}\big]\big\}dt' = -\frac{d}{dt}\rho^S_{00}(t),
\end{aligned} \tag{58}$$

where for simplicity we dropped the $z$-dependence of these functions that have non-zero values only for $z = 0$. These equations describe a coupling of the polarization with the population depending on the coherent field components $\mathcal{E}(t)$ and $\mathcal{E}_2(t)$ and the matrix elements of the self-consistent field fluctuations of the environment particles $\zeta_{01}$. In (57) we used the notation

$$\gamma_\perp = \lambda_{01} + \lambda_{10} + \lambda_{00} + \lambda_{11} \tag{59}$$

for the damping rate of polarization. For the population difference with the normalization condition

$$\rho_{11}(t) - \rho_{00}(t) = \langle N \rangle_t \tag{60a}$$

$$\rho_{11}(t) + \rho_{00}(t) = 1, \tag{60b}$$

we obtain the decay term of the form

$$-4\big[\lambda_{01}\rho_{11}^S(t) - \lambda_{10}\rho_{00}^S(t)\big] = -\gamma_{\parallel}\big[\langle N \rangle_t - w_T\big], \tag{61}$$

with the decay rate

$$\gamma_{\parallel} = 2(\lambda_{01} + \lambda_{10}) \tag{62}$$

and an equilibrium asymptotic solution:

$$w_T = -\frac{\lambda_{01} - \lambda_{10}}{\lambda_{01} + \lambda_{10}}. \tag{63}$$

With (22), or the simpler relations (4)-(5), and (6), one obtains these parameters as functions of the environment characteristics: interaction potentials, densities of states, occupation probabilities, temperature. From these expressions, we notice that the decay rate $\gamma_{\parallel}$ and the equilibrium population difference $w_T$ depend only on non-diagonal elements of the potential matrix, i.e. on correlated transitions of the system and environment particles. The incoherent relaxation rate $\gamma_{\perp}$ includes also diagonal elements of the interaction potential, i.e. processes without energy change, that is in agreement with the phenomenological understanding of this coefficient.

In Eq. (57) we distinguish two terms in the difference $\zeta_{11} - \zeta_{00}$ of the self-consistent field fluctuations diagonal matrix elements: (1) a first-order Markovian term describing a shift of the resonance frequency, and (2) a second-order non-Markovian term describing a spectrum modification from the Lorentzian shape due to the fluctuations of the energy levels. We notice also terms in the non-diagonal matrix elements $\zeta_{10}$ of the self-consistent field fluctuations: (1) a first-order Markovian term describing transitions stimulated by the self-consistent field of the environment, and (2) a non-Markovian term describing a modification of the spectrum from the Lorentzian shape due to these transitions. From the two equations (57) and (58), we notice that the self-consistent field of the environment with the amplitude $\zeta_{10}$ is somehow similar to the electromagnetic field in realizing a coupling of the non-diagonal and diagonal elements of the density matrix. Non-Markovian terms in the environment self-consistent field amplitude $\zeta_{10}$ arise in the polarization and also in the population equations.

For the system of non-linear equations (57)-(58), we consider a solution of the form (52) for the polarization and a similar expression for the population:

$$\rho_{11}^S(t) - \rho_{00}^S(t) \approx w(t) + w_1(t)e^{-i\omega t} + w_1^*(t)e^{i\omega t} + w_2(t)e^{-i2\omega t} + w_2^*(t)e^{i2\omega t}. \tag{64}$$

We obtain equations of slowly-varying amplitudes:

$$\frac{d}{dt}\mathcal{S}_0(t) = -[\gamma_\perp + i(\omega_0 + \zeta_{11} - \zeta_{00})]\mathcal{S}_0(t) + i\big\{2\zeta_{10}w(t) + + g\big[\mathcal{E}(t)w_1^*(t) - \mathcal{E}^*(t)w_1(t) + \frac{1}{2}\big(\mathcal{E}_2(t)w_2^*(t) - \mathcal{E}_2^*(t)w_2(t)\big)\big]\big\} \tag{65a}$$

$$\frac{d}{dt}\mathcal{S}(t) = -[\gamma_\perp + i(\omega_0 + \zeta_{11} - \zeta_{00} - \omega)]\mathcal{S}(t) + i\big\{2\zeta_{10}w_1(t) + + ig\big[\mathcal{E}(t)w(t) - \mathcal{E}^*(t)w_2(t) + \frac{1}{2}\mathcal{E}_2(t)w_1^*(t)\big]\big\} + + \int_0^t \big[(\zeta_{11} - \zeta_{00})^2\mathcal{S}(t') - 2\zeta_{10}(\zeta_{11} - \zeta_{00})w_1(t')\big]e^{i(\omega-\omega_0)(t-t')}dt' \tag{65b}$$

$$\frac{d}{dt}\mathcal{S}_2(t) = -[\gamma_\perp + i(\omega_0 + \zeta_{11} - \zeta_{00} - 2\omega)]\mathcal{S}_2(t) + + i\big\{2\zeta_{10}w_2(t) + g\big[\mathcal{E}(t)w_1(t) + \frac{1}{2}\mathcal{E}_2(t)w(t)\big]\big\} \tag{65c}$$

$$\frac{d}{dt}w(t) = -\gamma_\parallel[w(t) - w_T] - - i\frac{g}{2}\big\{\mathcal{E}(t)\mathcal{S}^*(t) - \mathcal{E}^*(t)\mathcal{S}(t) + \frac{1}{2}\big[\mathcal{E}_2(t)\mathcal{S}_2^*(t) - \mathcal{E}_2^*(t)\mathcal{S}_2(t)\big]\big\} \tag{65d}$$

$$\frac{d}{dt}w_1(t) = -(\gamma_\parallel - i\omega)w_1(t) + i\zeta_{10}\mathcal{S}(t) - - ig\big\{\mathcal{E}(t)\mathcal{S}_0(t) + \frac{1}{2}\big[\mathcal{E}_2(t)\mathcal{S}^*(t) - \mathcal{E}^*(t)\mathcal{S}_2(t)\big]\big\} + + \int_0^t \big[2\zeta_{10}^2 w_1(t') - \zeta_{10}(\zeta_{11} - \zeta_{00})\mathcal{S}(t')\big]e^{i(\omega-\omega_0)(t-t')}dt' \tag{65e}$$

$$\frac{d}{dt}w_2(t) = -(\gamma_\parallel - i2\omega)w_2(t) - i\frac{g}{2}\big[\mathcal{E}(t)\mathcal{S}(t) + \mathcal{E}_2(t)\mathcal{S}_0(t)\big] \tag{65f}$$

As the conventional Bloch-Feynman equations, these equations describe the polarization-population coupling by a quasi-resonant electromagnetic field $\mathcal{E}(t)$, but taking into account the matrix elements $\zeta_{00}, \zeta_{11}$ and $\zeta_{10}$ of the fluctuations of the self-consistent field of the environment particles. This field leads to a zero-order polarization, non-Markovian terms, and additional couplings with the higher-order harmonics. In this system we distinguish the two Bloch-Feynman-type equations (65b) and (65d) of the polarization amplitude $\mathcal{S}(t)$ and of the population $w(t)$ respectively. We notice that the population equation (65d) is a mere generalization of the conventional Bloch-Feynman population equation by including the second-order harmonics. New effects are described by the polarization equation (65b): (1) a transition frequency shift $\zeta_{11} - \zeta_{00}$, and (2) a non-Markovian term depending on this shift and on the fluctuations $\zeta_{10}$ of the self-consistent field. Eq. (65e) describes the generation of a first-order harmonics of the population by the coupling of this term to the coherent field $\mathcal{E}(t)$, to the polarization $\mathcal{S}(t)$ and to the higher-order polarization terms. From Eq. (65f), we notice that the second-order term of the population is also coupled to the polarization $\mathcal{S}(t)$ by the injected electric field $\mathcal{E}(t)$. Finally, from Eq. (65a) we notice that a zero-order polarization arises due to the coupling to the fluctuations of the self-consistent field $\zeta_{10}$ and to the higher-order harmonics of the population by the injected electric field $\mathcal{E}(t)$ and by the higher-order harmonics of this field generated by the non-linearity of the system.

## VI. NON-MARKOVIAN MAXWELL-BLOCH EQUATIONS AND ABSORPTION SPECTRUM

Equations (65) describe complex dissipative phenomena, including fluctuations of the energy levels and transitions between these levels with non-Markovian effects that arise from the fluctuations of the self-consistent field of the environment. However, besides the well-known decay processes, in the following we take into account only the non-Markovian effects due to the fluctuations of the energy levels. For a single propagating wave $\mathcal{E}(z,t) = \mathcal{E}_+(z,t)$, of amplitude $\mathcal{E}(t) \equiv \mathcal{E}(z,t)\mid_{z=0}$ at the coordinate $z = 0$ of the two-level system, from (65) we obtain a non-Markovian term in addition to the well-known Maxwell-Bloch equations:

$$\frac{d}{dt}\mathcal{S}(t) = -\gamma_\perp(1 - i\delta\omega)\mathcal{S}(t) + ig\mathcal{E}(t)w(t) + \gamma_n^2 \int_0^t \mathcal{S}(t')e^{i(\omega-\omega_0)(t-t')}dt' \tag{66a}$$

$$\frac{d}{dt}w(t) = -\gamma_\parallel[w(t) - w_T] - i\frac{g}{2}[\mathcal{E}(t)\mathcal{S}^*(t) - \mathcal{E}^*(t)\mathcal{S}(t)] \tag{66b}$$

$$\frac{d}{dt}\mathcal{E}(t) = \frac{\partial}{\partial t}\mathcal{E}(z,t)\mid_{z=0} + c\frac{\partial}{\partial z}\mathcal{E}(z,t)\mid_{z=0} = -iG\mathcal{S}(t). \tag{66c}$$

Since we are primarily interested in the interaction of an electromagnetic mode with an atomic system, the field dissipation is neglected. In (66a)

$$\gamma_n \equiv \zeta_{11} - \zeta_{00} \tag{67}$$

is the shift of the transition frequency of the system induced by the fluctuations of the self-consistent field of the environment that, at the same time, is the non-Markovian coefficient, and

$$\delta\omega = \frac{\omega - \omega_0 - \gamma_n}{\gamma_\perp} \tag{68}$$

is the relative frequency detuning.

Although the non-Markovian Maxwell-Bloch equations have been derived by a truncation of perturbation series, they guarantee the positivity of the density matrix

$$\rho_{11}(t) = \frac{1 + w(t)}{2} > 0 \ , \qquad \rho_{00}(t) = \frac{1 - w(t)}{2} > 0 \ , \tag{69}$$

that means

$$-1 < w(t) < 1 \ . \tag{70}$$

Really, if at a certain time $t_0$ the density matrix is positive, it can be diagonalized:

$$\frac{d}{dt}\mathcal{S}(t)|_{t=t_0} = ig\mathcal{E}(t_0)w(t_0) + \gamma_n^2 \int_0^{t_0} \mathcal{S}(t')e^{i(\omega-\omega_0)(t_0-t')}dt' \tag{71a}$$

$$\frac{d}{dt}w(t)|_{t=t_0} = -\gamma_\parallel[w(t_0) - w_T]. \tag{71b}$$

Thus, for

$$\begin{aligned} w(t_0) > w_T \quad &\Rightarrow \quad \frac{d}{dt}w(t)|_{t=t_0} < 0, \quad \text{and for} \\ w(t_0) < w_T \quad &\Rightarrow \quad \frac{d}{dt}w(t)|_{t=t_0} > 0, \end{aligned} \tag{72}$$

that means that $w(t)$ approaches to $w_T$. Remaining positive in a very short interval of time, the density matrix can be diagonalized again, i.e. it remains positive in a following short interval of time. Thus, the positivity of the density matrix is preserved.

We separate the real and imaginary parts of polarization and field

$$\mathcal{S}(t) = u(t) - iv(t) \tag{73a}$$

$$\mathcal{E}(t) = \mathcal{F}(t) + i\mathcal{G}(t), \tag{73b}$$

and define the differential functions of polarization and population:

$$L_\mathcal{S}(t) \equiv \frac{1}{\gamma_\perp}\frac{1}{\mathcal{S}(t)}\frac{d}{dt}\mathcal{S}(t) \equiv L'_\mathcal{S}(t) - iL''_\mathcal{S}(t) \tag{74a}$$

$$L_w(t) \equiv \frac{1}{\gamma_\parallel}\frac{1}{w(t)}\frac{d}{dt}w(t). \tag{74b}$$

With the integral function

$$I_{\mathcal{S}} = \frac{\gamma_n^2}{\gamma_\perp}\frac{1}{\mathcal{S}(t)}\int_0^t \mathcal{S}(t')e^{i(\omega-\omega_0)(t-t')}dt' \equiv I'_{\mathcal{S}}(t) - iI''_{\mathcal{S}}(t), \tag{75}$$

in Eq. (66a) we get the integral-differential function of polarization:

$$\sigma(t) \equiv L_{\mathcal{S}}(t) - I_{\mathcal{S}}(t) \equiv \sigma_1(t) - i\sigma_2(t). \tag{76}$$

With (73a), (74) and (75), we get the real differential functions of polarization and of population

$$L'_{\mathcal{S}}(t) = \frac{1}{\gamma_\perp}\frac{d}{dt}\ln[u^2(t)+v^2(t)]^{1/2} \tag{77a}$$

$$L''_{\mathcal{S}}(t) = \frac{1}{\gamma_\perp}\frac{d}{dt}\arctan\frac{v(t)}{u(t)} \tag{77b}$$

$$L_w(t) = \frac{1}{\gamma_\parallel}\frac{d}{dt}\ln|w(t)| \tag{77c}$$

and the real integral functions of polarization:

$$I'_{\mathcal{S}}(t) = \frac{\gamma_n^2}{\gamma_\perp}\int_0^t \left\{\frac{u(t)u(t')+v(t)v(t')}{u^2(t)+v^2(t)}\cos[(\gamma_\perp\delta\omega+\gamma_n)(t-t')] + \right. \tag{78a}$$

$$\left. + \frac{u(t)v(t')-v(t)u(t')}{u^2(t)+v^2(t)}\sin[(\gamma_\perp\delta\omega+\gamma_n)(t-t')]\right\}dt'$$

$$I''_{\mathcal{S}}(t) = \frac{\gamma_n^2}{\gamma_\perp}\int_0^t \left\{\frac{u(t)v(t')-v(t)u(t')}{u^2(t)+v^2(t)}\cos[(\gamma_\perp\delta\omega+\gamma_n)(t-t')] - \right. \tag{78b}$$

$$\left. - \frac{u(t)u(t')+v(t)v(t')}{u^2(t)+v^2(t)}\sin[(\gamma_\perp\delta\omega+\gamma_n)(t-t')]\right\}dt'.$$

Thus, we obtain the real integral-differential functions of polarization

$$\sigma_1(t) = L'_{\mathcal{S}}(t) - I'_{\mathcal{S}}(t) \tag{79a}$$

$$\sigma_2(t) = L''_{\mathcal{S}}(t) - I''_{\mathcal{S}}(t) \tag{79b}$$

and the expression of population (77c) as a solution of the non-Markovian Maxwell-Bloch equations of the slowly-varying amplitudes:

$$\frac{d}{dt}u(t) = \qquad -\gamma_\perp u(t) + \gamma_\perp \delta\omega v(t) - g\mathcal{G}(t)w(t) + I_u(t), \tag{80a}$$

$$I_u(t) \equiv \gamma_n^2 \int_0^t \big\{u(t')\cos[(\gamma_\perp\delta\omega + \gamma_n)(t-t')] + v(t')\sin[(\gamma_\perp\delta\omega + \gamma_n)(t-t')]\big\}dt'$$

$$\frac{d}{dt}v(t) = \qquad -\gamma_\perp v(t) - \gamma_\perp \delta\omega u(t) - g\mathcal{F}(t)w(t) + I_v(t), \tag{80b}$$

$$I_v(t) \equiv \gamma_n^2 \int_0^t \big\{v(t')\cos[(\gamma_\perp\delta\omega + \gamma_n)(t-t')] - u(t')\sin[(\gamma_\perp\delta\omega + \gamma_n)(t-t')]\big\}dt'$$

$$\frac{d}{dt}w(t) = -\gamma_\parallel[w(t) - w_T] + g[\mathcal{G}(t)u(t) + \mathcal{F}(t)v(t)] \tag{80c}$$

$$\frac{d}{dt}\mathcal{F}(t) = \frac{\partial}{\partial t}\mathcal{F}(z,t)\mid_{z=0} + c\frac{\partial}{\partial z}\mathcal{F}(z,t)\mid_{z=0} = -Gv(t) \tag{80d}$$

$$\frac{d}{dt}\mathcal{G}(t) = \frac{\partial}{\partial t}\mathcal{G}(z,t)\mid_{z=0} + c\frac{\partial}{\partial z}\mathcal{G}(z,t)\mid_{z=0} = -Gu(t). \tag{80e}$$

The non-Markovian dynamics is described by the two slowly-varying time-integrals $I_u(t)$, $I_v(t)$.

We notice that in very short intervals of time, when dissipation is negligible, these equations satisfy conservation relations of the Bloch vector

$$\frac{d}{dt}[u^2(t) + v^2(t) + w^2(t)] = 0, \tag{81}$$

and of the electromagnetic energy production in the quantization volume $\mathcal{V}$:

$$\frac{d}{dt}\big\{\hbar\omega_0\rho_{11}(t) + \mathcal{V}\frac{1}{2}\varepsilon[\mathcal{F}^2(t) + \mathcal{G}^2(t)]\big\} = 0, \tag{82}$$

where we used the relation $\rho_{11}(t) = \frac{1}{2}[1+w(t)]$, obtained from (60) when the rapid oscillations of population are neglected: $\langle N\rangle_t \approx w(t)$. If this system is an element of an assembly of $\mathcal{N}$ systems in a volume unit, $\mathcal{V} = 1/\mathcal{N}$.

From Eqs. (66) with (74)-(79), we get the polarization and population

$$\mathcal{S}(t) = \frac{igw(t)\mathcal{E}(t)}{\gamma_\perp\{1 + \sigma_1(t) - i[\delta\omega + \sigma_2(t)]\}} \tag{83a}$$

$$w(t) = \frac{(1+\delta\omega^2)w_T}{[1 + L_w(t)](1+\delta\omega^2) + g^2\frac{|\mathcal{E}(t)|^2}{\gamma_\parallel\gamma_\perp}\eta(t)}, \tag{83b}$$

entirely including the time-evolution of the system in the functions $\sigma_1(t), \sigma_2(t), L_w(t)$ and

$$\eta(t) = \frac{1 + \sigma_1(t)}{1 + \frac{1}{1+\delta\omega^2}\left[\sigma_1^2(t) + \sigma_2^2(t) + 2\sigma_1(t) + 2\delta\omega\sigma_2(t)\right]}. \tag{84}$$

With these expressions, from the field equation (66c) we obtain the time-dependent absorption coefficient:

$$\begin{aligned}\alpha(t) \equiv & -\frac{1}{|\mathcal{E}(z,t)|}\frac{d}{cdt}|\mathcal{E}(z,t)|\,|_{z=0}= \\ = & -\frac{Gg}{c\gamma_\perp}\frac{1+\sigma_1(t)}{[1+\sigma_1(t)]^2+[\delta\omega+\sigma_2(t)]^2}w(t).\end{aligned} \tag{85}$$

This dynamic absorption coefficient has a modified form compared with the Lorentzian absorption coefficient

$$\alpha_L = -\frac{Gg}{c\gamma_\perp}\frac{1}{1+\delta\omega^2}w_L. \tag{86}$$

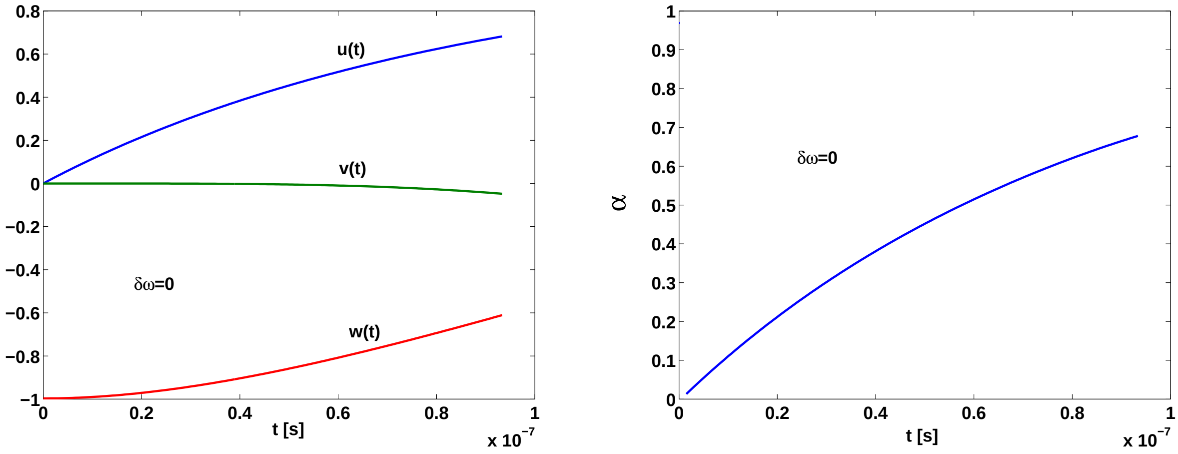
In (85) the integral-differential functions of time $\sigma_1(t), \sigma_2(t)$ and the population function of time $w(t)$ appear. The time-dependent population function (83b) is a modified form of the well-known solution of the stationary Maxwell-Bloch equations

$$w_L = \frac{(1+\delta\omega^2)w_T}{1+\delta\omega^2+g^2\frac{|\mathcal{E}|^2}{\gamma_\parallel\gamma_\perp}}, \tag{87}$$

with the differential function $L_w(t)$ and the integral-differential function $\eta(t)$. The expression (86) with (87) includes the saturation effect in comparison with the absorption spectrum at a low excitation $g^2|\mathcal{E}|^2 \ll \gamma_\parallel\gamma_\perp$: $\alpha_0 = \alpha_L(w_L \to w_T)$.

## VII. APPLICATION

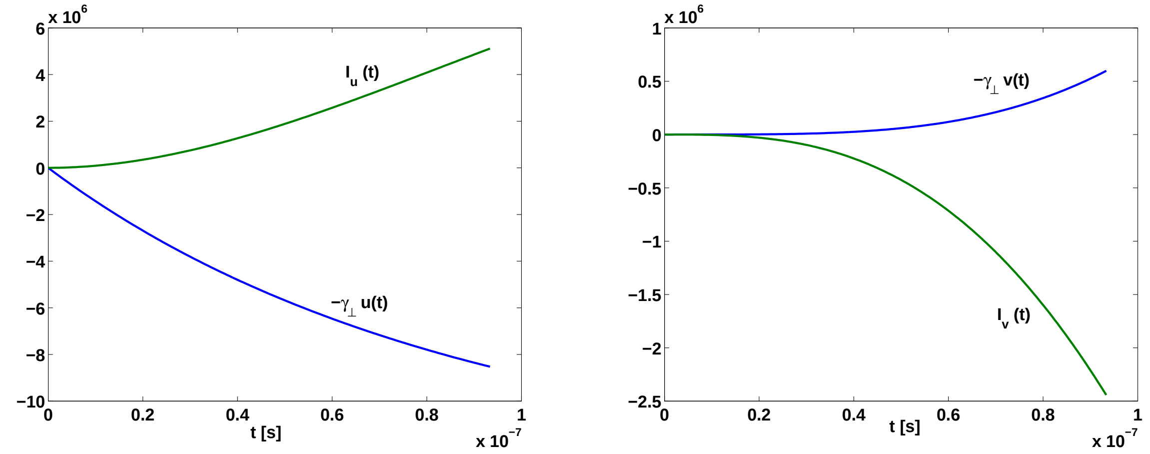
As an application, we consider a double array of $10^{16}$ quantum dots/m$^2$ situated at a semiconductor p-i-n junction on the two sides of the i-layer [23]. A coherent electromagnetic field incident on this array is absorbed by transitions of the electrons tunneling through the thin i-layer, from the upper margin of the valence band of the p-region to the lower margin of the conduction band of the n-region. Considering equal concentrations of acceptors and donors $N_A = N_D = 8.944 \times 10^{16}$ cm$^{-3}$ in the two regions n and p, for GaAs at temperature $T = 100\ ^0$C, we obtain a transition energy $\hbar\omega_0 = 0.2056$ eV. As dissipative coefficients we consider $\gamma_\parallel = 0.50084 \times 10^7$ s$^{-1}$, $\gamma_\perp = \gamma_n = 1.25 \times 10^7$ s$^{-1}$ and $w_T = -0.99667$. We study the dynamics of this device under the action of a coherent electromagnetic wave of $S = 100$ W/m$^2$ applied as a step at the moment of time $t = 0$. At the initial moment $t = 0$ the system observables still have their equilibrium mean-values $u(0) = 0$, $v(0) = 0$, $w(0) = w_T$. After this moment, a time evolution begins (Fig. 1 *a*)), while an absorption




$(a)$ $(b)$

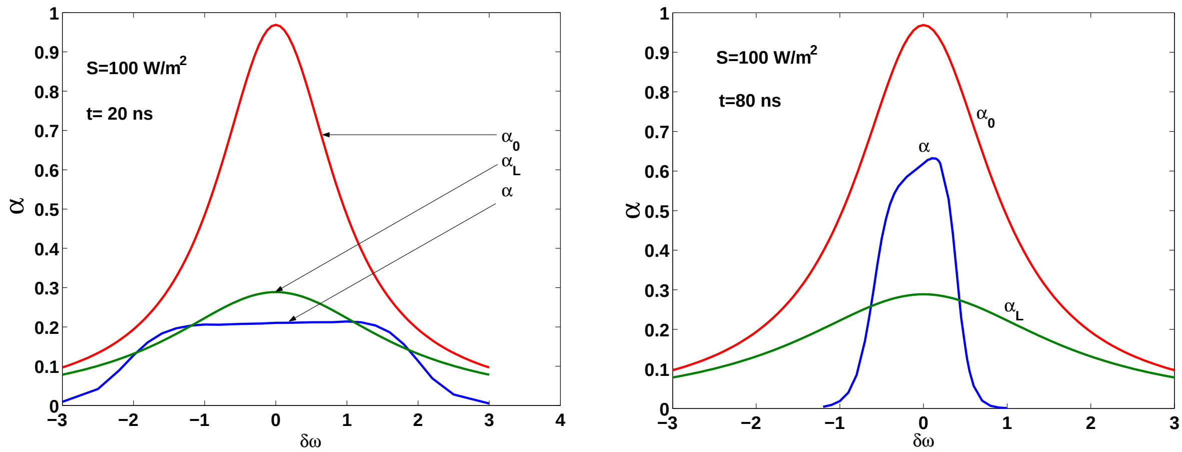
FIG. 1: Non-Markovian evolution of an array of two-level quantum dots under the action of a resonant electromagnetic mode: a) the polarization $u(t), v(t)$ and the population $w(t)$, b) the dynamic absorption coefficient $\alpha(t)$.

coefficient arises (Fig. 1 $b$)). Here we considered a perfect resonance ($\delta\omega = 0$), when the the coupling of the two polarization components $u(t)$ and $v(t)$ is entirely caused by the non-Markovian integrals $I_u(t), I_v(t)$ in the polarization equations (80a) and (80b). With a phase choice, the initial field amplitude is of a simpler form $\mathcal{E}(0) = i\mathcal{G}(0)$, according to the field equation (80e) corresponding to the polarization component $u(t)$. A polarization component $v(t)$ and a field component $\mathcal{F}(t)$ arise according to the polarization-field equations (80b) and (80d) only due the non-Markovian integrals $I_u(t)$ and $I_v(t)$ of the two polarization equations (80a) and (80b) respectively. From Fig. 2 we notice that these terms are competitive with the Markovian ones $\gamma_\perp u(t)$ and $\gamma_\perp v(t)$. For short times $t$, these terms describe mainly modifications of the decay rates of the two polarization components, corresponding to the large values of the coefficients in $\cos[\gamma_n(t-t')]$, while the non-Markovian coupling between these components is weak due to the small values of $\sin[\gamma_n(t-t')]$. Only for longer times $t$ this coupling increases, leading also to a significant dephasing of the electromagnetic field, given by $\arctan\frac{\mathcal{F}(t)}{\mathcal{G}(t)}$, as a non-Markovian effect. Performing these calculations for certain time values $t = 20,\ 80$ ns, and different frequency detunings $\delta\omega$, we obtained the absorption spectra in Fig. 3. For a rather short time, while the two non-Markovian integrals $I_u(t)$ and $I_v(t)$ remain rather small, the dynamic spectrum $\alpha$ remains rather wide and of an amplitude not larger than the amplitude of the stationary spectrum $\alpha_L$ (Fig. 3 $a$)). When




$(a)$ $(b)$

FIG. 2: Non-Markovian terms in comparison with the Markovian decay terms of the two polarization equations.




$(a)$ $(b)$

FIG. 3: Non-Markovian dynamic absorption spectrum in comparison with the Lorentzian spectra $\alpha_0$ at low excitation field, and $\alpha_L$ including the saturation effect: $a$) for small values of the non-Markovian integrals ($t = 20$ ns); $b$) for large values of the non-Markovian integrals ($t = 80$ ns).

the time increases and the non-Markovian integrals take significant values, the dynamic absorption spectrum becomes narrower and has a larger amplitude (Fig. 3 $b$)). In Fig. 4 we represent the dynamic spectrum $\alpha$ and the corresponding integral-differential functions $\sigma_1$ and $\sigma_2$. According to (85), we notice that the integral-differential function $\sigma_2$ is an additional nonlinear dephasing, responsible for the rather sudden limits of the dynamic spectrum in comparison with the much slower limits of the Lorentzian spectrum. The integral-differential

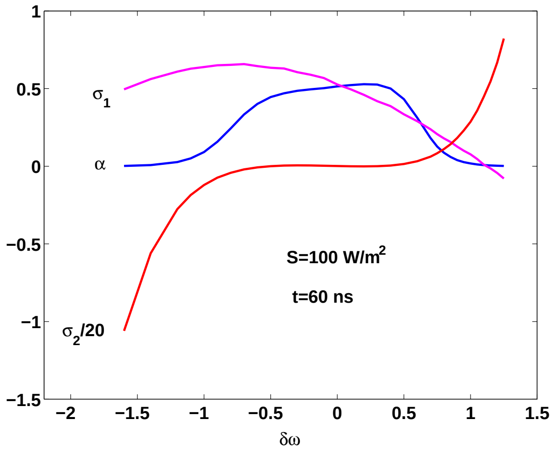



FIG. 4: Absorption dynamic spectrum $\alpha$ and the corresponding integral-differential functions $\sigma_1, \sigma_2$.

function $\sigma_1$ is responsible for the increase of the amplitude of the dynamic spectrum by diminishing the saturation effect according to (83b) and (84).

## VIII. SUMMARY AND CONCLUSIONS

Using a series expansion of the weak dissipative coupling, we derived a second-order non-Markovian quantum master equation for a system of charged Fermions. In a previous approach [18], exactly in the same approximation we obtained a Markoovian master equation only considering that the self-consistent field of the environment particles can be taken into account by a simple diagonalization of the Hamiltonian. However this is not exactly correct, because the environment particles at a certain temperature fluctuate, leading to fluctuations of the energy levels of the system and to stimulated transitions between these levels. Here we showed that these fluctuations lead also to an additional non-Markovian term in the master equation. Consequently, the dissipative dynamics of a system of Fermions has three major components: 1) a Markovian component due to the correlated transitions of the system and the environment particles, leading to relaxation processes, 2) a Markovian component due to the fluctuations of the self-consistent field that determines fluctuations of the energy levels and transitions between these levels, and 3) a non-Markovian component due to the time evolution of the self-consistent field of the environment that, in this way, behaves as a memory environment. Being derived by a truncation of perturbations series, the resultant quantum master equation is valid for rather short intervals of time, while for longer intervals of time it could violate the positivity of the density matrix. In this approach the influence of the system on the environment is taken into account. The only simplifying hypothesis used in our approach consists in a large density of the environment states. This condition is sufficient to derive the quantum master equation for a system under the influence of the environment. This equation, as the system master equation, is in agreement with the detailed balance principle. Being derived by a truncation of the perturbation series, the quantum master equation of the fermion environment coupled only with the system of interest may describe far from equilibrium states, and finally violate the positivity of the density matrix. However, if a stronger coupling with a bigger environment is taken into account, the primary environment remains in quasi-equilibrium states, while the positivity of the density matrix is preserved. We considered the interaction with an electromagnetic field and derived non-Markovian Maxwell-Bloch equations. It is remarkable that the additional non-Markovian term does not alter the positivity of the density matrix. We calculated the dynamic absorption spectrum of an array of two-level quantum dots and discussed the non-Markovian effects. These effects are described by integral-differential functions, in some conditions strongly narrowing the absorption spectrum and diminishing the system saturation.

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