Open Quantum Systems of Particles and Principle 2 of Thermodynamics

Eliade Stefanescu[1]

[1] Center of Advanced Studies in Physics
at the Institute of Mathematics "Simion Stoilow" of the Romanian Academy,
21 Calea Grivitei, 010702 Bucharest, Romania

E-mail: eliadestefanescu@yahoo.fr

Abstract. We consider quantum master equations for a system of Fermions and an electromagnetic field, and apply these equations to a superradiant semiconductor structure converting environmental heat into coherent electromagnetic energy. For a non-irradiative system, these equations describe a time evolution with entropy increase, according to principle 2 of thermodynamics. However, for a superradiant system, the entropy may decrease, while the asymptotic solution of these equations, corresponding to constant entropy, describes a field radiation on the account of environmental heat absorption by Peltier effect.

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1. Introduction

Physics of open quantum systems has a long history [1, 2, 3, 4, 5]. A general method, for deriving a quantum master equation for a certain system with the density matrix $\rho(t)$, in a dissipative environment with the density matrix R, has been disclosed by Ford, Lewis, and O'Connell in [6]. In a previous paper [7], we derived a non-Markovian master equation for a system of Fermions in a complex dissipative environment of other Fermions, Bosons, and the free electromagnetic field. This equation includes a Markovian term, which describes correlated transitions of the system and environment particles, with energy conservation. For a system of electrons coupled to a coherent electromagnetic field, when the self-consistent field matrix elements are considered for the two-body potential between the system and environment particles, a thermal fluctuation Hamiltonian term, and a time non-local term, are also obtained in the second-order approximation [7].

We applied this equation to a super radiant semiconductor heterostructure converting environmental heat into coherent electromagnetic energy, we called quantum heat converter [8], working on the basis of a new kind of active quantum dots, we call quantum injection dots [9]. In this paper we discuss the agreement of the device operation with principle 2 of thermodynamics, which asserts that, in an isolated system, an ordering process is forbidden. However, there is a problem that this principle seems to be in contradiction with the highly ordered structure of the planet and, consequently, of the planetary system we live in, which is isolated from the rest of our galaxy [10]. In section 2, we describe the dissipative quantum dynamics of a superradiant semiconductor heterostructure by three quantum master equations, for the system of active quantum dots, for the electromagnetic field, and for the crystal lattice vibrations. In section 3, we describe the operation of a quantum heat converter. We derive an entropy equation, and show that for a superradiant system the entropy may decrease. In section 4, we give conclusions.

2. Dissipative dynamics of a superradiant semiconductor structure

We consider an electromagnetic field with two counter-propagating modes, with the densities $\rho_+^F(x,t), \rho_-^F(x,t)$, interacting with an active system of electrons with the density $\rho^S(t)$, a system of quasi-free electrons/holes in the conduction regions with the densities of states $g_\alpha^F(\varepsilon)$ and the occupation probabilities $f_\alpha^F(\varepsilon)$ of these states of energies ε, and a system of phonons generated by this field, with counter-propagating modes with densities $\rho_+^P(x,t), \rho_-^P(x,t)$, which decay in the environment of conduction electrons/holes. With the creation-annihilation operators a_+^+ - a_+ and a_-^+ - a_- for the two field waves, and the Fermi operators c_i^+ - c_i for the active electrons, we obtain the field quantum master equations

$$\begin{aligned}
&\frac{d}{dt}\rho_+^F(x,t) = -\mathrm{i}\omega[a_+^+ a_+, \rho_+^F(x,t)] \qquad (1)\\
&+\sum_{j>i}\omega_{ji}\vec{K}_A\vec{r}_{ij}\left[\langle c_j^+ c_i\rangle a_+ e^{\mathrm{i}kx} - \langle c_i^+ c_j\rangle a_+^+ e^{-\mathrm{i}kx}, \rho_+^F(x,t)\right]\\
&+\mathrm{i}\frac{\omega}{2\omega_\nu}Kv\left[\langle a_{\nu+}^+\rangle a_+ + \langle a_{\nu+}\rangle a_+^+, \rho_+^F(x,t)\right]\\
&+\Lambda_x\int_0^x \left\{[a_+\rho_+^F(x',t'), a_+^+] + [a_+, \rho_+^F(x',t')a_+^+]\right\} e^{-\mathrm{i}k(x-x')}dx'
\end{aligned}$$

$$\begin{aligned}
&\frac{d}{dt}\rho_-^F(x,t) = -\mathrm{i}\omega[a_-^+ a_-, \rho_-^F(x,t)] \qquad (2)\\
&+\sum_{j>i}\omega_{ji}\vec{K}_A\vec{r}_{ij}\left[\langle c_j^+ c_i\rangle a_- e^{\mathrm{i}kx} - \langle c_i^+ c_j\rangle a_-^+ e^{-\mathrm{i}kx}, \rho_+^F(x,t)\right]\\
&+\mathrm{i}\frac{\omega}{2\omega_\nu}Kv\left[\langle a_{\nu-}^+\rangle a_- + \langle a_{\nu-}\rangle a_-^+, \rho_-^F(x,t)\right]\\
&+\Lambda_x\int_x^{L_D} \left\{[a_-\rho_-^F(x',t'), a_-^+] + [a_-, \rho_-^F(x',t')a_-^+]\right\} e^{-\mathrm{i}k(x'-x)}dx',
\end{aligned}$$

where ω is the field frequency, ω_{ji} are transition quantum dot transition frequencies, $\vec{K}_A = \vec{1}_y\sqrt{\alpha\frac{\lambda}{\mathcal{V}_A}}$ is a vector in the field polarization direction $\vec{1}_y$, depending on the spectroscopic constant α, the field wavelength λ, and the quantization volume of a quantum dot $\mathcal{V}_A$, we call quantization vector, $K = \sqrt{\alpha\frac{\lambda}{\mathcal{V}}}$ is the quantization number of the electromagnetic field in a quantization volume $\mathcal{V}$, $a^{+}_{\nu+}$ - $a_{\nu+}$ are phonon creation-annihilation operators, ω_ν is the frequency of these phonons generated by the electromagnetic field with the same wave number $k_\nu = k$, v is the sound velocity in the crystal, Λ_x is the field dissipation constant, and L_D is the thickness of the active semiconductor structure, between the two mirrors including this structure. The field master equations (1)-(2) are space non-local, the field quantization volume $\mathcal{V}$ being much larger than the field wavelength λ. This electromagnetic field excites a vibration field described by similar master equations, with the phonon dissipation constant Λ^P_x. For a description of the whole system of interest, besides the master equations (1)-(2) of the electromagnetic field, and of the crystal vibrations induced by this field, we consider a master equation for the active system of electrons [8],

$$\frac{d}{dt}\rho^S(t) = -\frac{\mathrm{i}}{\hbar}[H, \rho^S(t)] + \sum_{ij} \lambda_{ij} \left\{[c^+_i c_j \rho^S(t), c^+_j c_i] + [c^+_i c_j, \rho^S(t) c^+_j c_i]\right\}, \qquad (3)$$

depending on the Hamiltonian $H = \sum_i \varepsilon_i c^+_i c_i + V$ of these electrons with the electron-field potential V, while λ_{ij} are decay/excitation rates. In [8], explicit expressions of these rates are derived for a semiconductor heterostructure.

3. Entropy equation for a superadiant dissipative system

We consider a super radiant semiconductor device absorbing heat from the environment [8] (figure 1). When an electron current I is injected in the device, an electromagnetic field is generated by quantum transitions at points of maximum field, while the electrons are thermally excited on deep-level paths at the nodes of this field in a Fabry-Perot cavity. Really, the electron current I enhances the lower states of a deep-level path, while the higher states of this path are depleted. In this way, a thermal junction becomes colder, attracting environmental heat, which excites the electrons on the deep-level path of this junction. We define the entropy

$$S_{DLP}(t) = -\sum_k \rho^S_{kk}(t)\left[\ln \rho^S_{kk}(t) - \ln \rho^S_{kk}(\infty)\right], \qquad (4)$$

for the many-level system of the deep-level path of a thermal junction, and

$$S_{SQD}(t) = -\rho^S_{00}(t) \ln \frac{\rho^S_{00}(t)}{\rho^S_{00}(\infty)} - \rho^S_{11}(t) \ln \frac{\rho^S_{11}(t)}{\rho^S_{11}(\infty)}. \qquad (5)$$

for a two-level superradiant quantum dot of a superradiant junction (see figure 1b). From the quantum master equations (1)-(3), we derive field, population, and polarization equations. We obtain two entropy equations:

$$\frac{d}{dt}S_{DLP}(t) = \sum_{ki} \lambda_{ki}\rho^S_{ii}(\infty)\left[\frac{\rho^S_{kk}(t)}{\rho^S_{kk}(\infty)} - \frac{\rho^S_{ii}(t)}{\rho^S_{ii}(\infty)}\right]\left[\ln \frac{\rho^S_{kk}(t)}{\rho^S_{kk}(\infty)} - \ln \frac{\rho^S_{ii}(t)}{\rho^S_{ii}(\infty)}\right], \qquad (6)$$

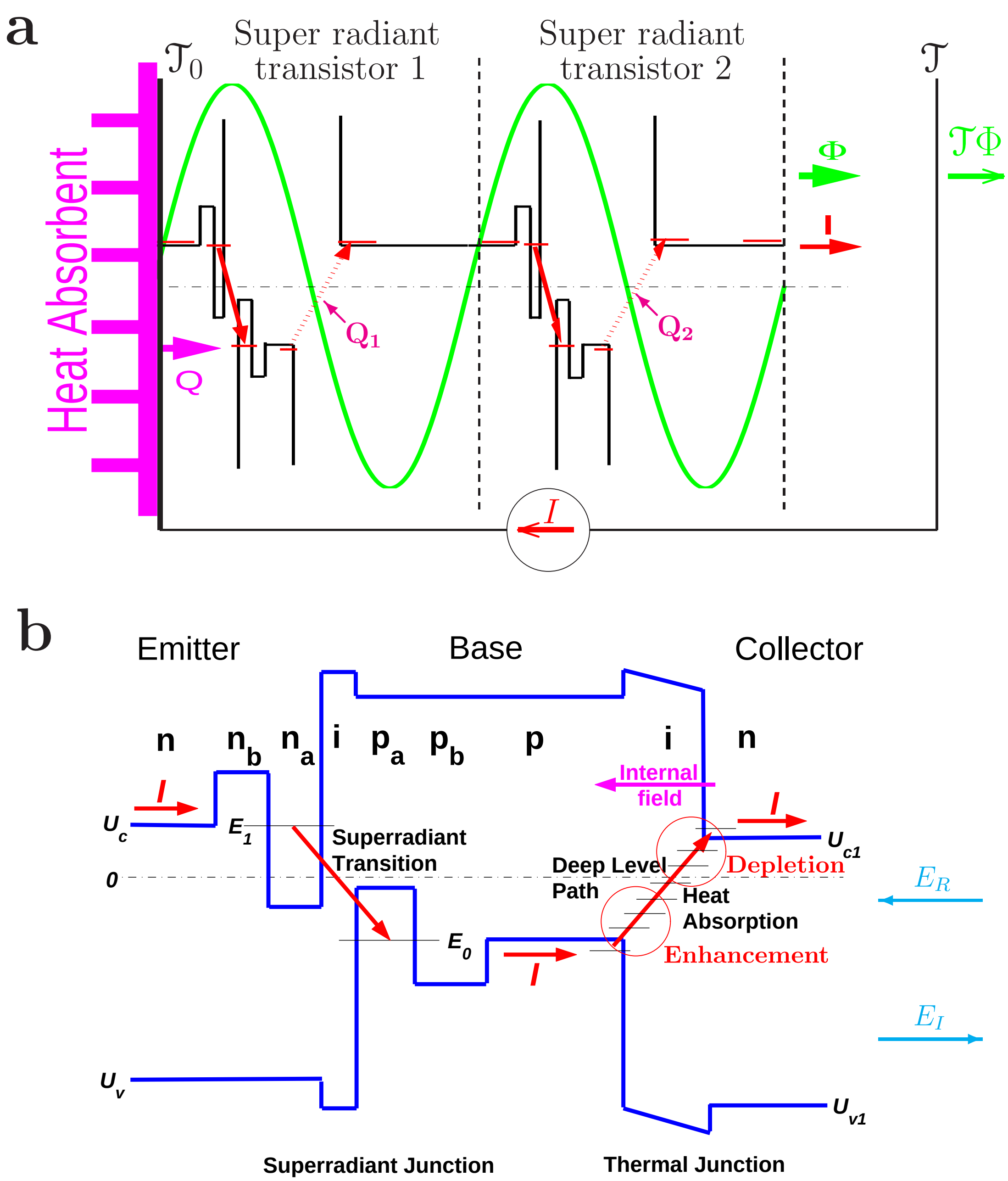

Figure 1. (a) Super radiant semiconductor device as a sequence of super radiant transistors, with super radiant quantum dots situated at points of maximum field, and deep-level paths at nodes of this field, absorbing heat from the environment $Q_1, Q_2, \ldots$; (b) Superradiant transistor: the electrons of a current I injected in the device by a potential difference $U_c - U_{c1}$ undergo a superradiant decay to a much lower potential $E_0 \ll E_1 \approx U_c$, in the superradiant junction, and a thermal excitation from E_0 to $U_{c1} \approx U_c$, in the thermal junction.

and

$$\frac{d}{dt}S_{SQD}(t) = \left\{ \frac{e^2}{4\hbar^2\omega^2 K_A^2} \left[\frac{d}{dt}|\mathcal{E}(t)|^2 + 2\gamma_F |\mathcal{E}(t)|^2 \right] \left[1 - \frac{\rho_{11}^S(t)}{\rho_{11}^S(\infty)} \right] \right.$$
$$\left. + 2\lambda_{10}\rho_{00}^S(\infty) \left[\frac{\rho_{11}^S(t)}{\rho_{11}^S(\infty)} - \frac{\rho_{00}^S(t)}{\rho_{00}^S(\infty)} \right] \right\} \left[\ln \frac{\rho_{11}^S(t)}{\rho_{11}^S(\infty)} - \ln \frac{\rho_{00}^S(t)}{\rho_{00}^S(\infty)} \right], \qquad (7)$$

where $\mathcal{E}(t)$ is the electric field amplitude of the superradiant electromagnetic field, and $\gamma_F = \frac{\Lambda_x + \Lambda_x^P}{4k}$ is the decay rate of this field. Equation (6) for the time-evolution of a many-level system of Fermions is a positively defined form, which corresponds to the so-called principle 2 of thermodynamics. Equation (7) for a superradiant two-level system includes a similar positively defined form for the two-level system, and a field dependent form with the sign given by the logarithmic factor. Really, the field dependent term in the curly bracket is always positive: when the population $\rho_{11}(t)$ increases to a higher asymptotic value $\rho_{11}(\infty)$, the field amplitude $|\mathcal{E}(t)|$ also increases; when the population $\rho_{11}(t)$ decays to a lower asymptotic value $\rho_{11}(\infty)$, the field amplitude $|\mathcal{E}(t)|$ decreases. In other words, when the superradiant field increases, the population $\rho_{11}(t)$ increases to a higher asymptotic value $\rho_{11}(\infty)$, while the population $\rho_{00}(t)$ decreases to a lower asymptotic value $\rho_{00}(\infty)$, i.e. the logarithmic factor is negative, which means that the field entropy term decreases. We notice that the field term in equation (7) dominates the atomic term, which being proportional to the excitation rate λ_{10} from the lower state $|0\rangle$ to the higher state $|1\rangle$, is very small.

4. Conclusions

We described a superradiant semiconductor structure by a system of five quantum master equations, for the two counter-propagating modes of the electromagnetic field, the two counter propagating modes of the crystal vibrations excited by this field, and the active quantum dots. For a many level system of Fermions we obtained an entropy increase, in agreement with principle 2 of thermodynamics, while for a superradiant system of Fermions, the entropy may decrease. For the steady-state, while the entropy is constant, a conversion of environmental heat into coherent electromagnetic energy is possible. On the account of this energy, other physical systems may evolve to states with lower entropy. In more general terms, a self-organization of a sufficiently large system may emerge by superradint processes taking place in this system.

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